

# Helical containers with classical and quantum fluids in rotating frame.

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We consider examples of the classical liquids confined by rotating helical boundaries and compare these examples with rotating helical reservoir filled by ultracold bosonic ensemble. From the point of view of observer who co-rotates with classical liquid trapped by reservoir the quantum fluid will move translationally alongside rotation axis while in laboratory frame the quantum fluid will stay in rest. This behavior of quantum ensemble which is exactly opposite to the classical case might be interpreted as a helical analog of Hess-Fairbank effect.

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## I. CLASSICAL DIELECTRIC FLOW AT ROOM TEMPERATURE IN HELICAL CHANNEL

Three examples of twisted flows in a rotating reference frame are  $\mu$  considered  $\mu/k_B T$  analytically. The first case is Stimulated Brillouin scattering in the liquids at the room temperature. Here incompressible Navier - Stokes liquid [1] is driven in rotation by helical interference pattern of the counter-propagating optical vortices with opposite angular momenta:

$$\rho \frac{\partial \vec{V}}{\partial t} + \rho(\vec{V} \cdot \nabla) \vec{V} = -\nabla p + \vec{F}_{ext}(\vec{r}, t) + \eta \Delta \vec{V}, \quad F_{ext}(\vec{r}, t) = -\nabla \delta p_{str}, \quad \delta p_{str} = -\frac{|\vec{E}|^2}{16\pi} \rho_0 \frac{\partial \epsilon}{\partial \rho} \quad (1)$$

where  $\vec{V}(\vec{r}, t)$  is a field of velocities,  $\rho(\vec{r}, t)$  is a liquid density,  $p$  is pressure,  $\eta$  is viscosity,  $F_{ext}(\vec{r}, t) = -\nabla \delta p_{str}$  is externally applied force which is ponderomotive in this particular case [2]. The electrostrictive pressure  $p_{str}$  pulls liquid into regions with a higher optical intensity having a form of the  $2\ell$  mutually embedded helices:

$$|E(z, r, \theta, t)|^2 = |E_f|^2 + |E_b|^2 \sim r^{2|\ell|} \exp \left[ -\frac{2r^2}{D_0^2(1 + z^2/k^2 D_0^4)} \right] [1 + \cos(2kz + 2\ell\theta + \delta\omega t)], \quad (2)$$

where the cylindrical coordinates  $\vec{r} = (z, r, \theta)$  are used,  $D_0$  is the radius of LG,  $\ell$  is vorticity,  $k = 2\pi/\lambda$  is wavenumber,  $\delta\omega$  is angular Doppler shift induced by rotation of the reference frame or emulated by rotation of Dove prism in phase conjugated setup [3]. When their frequencies detuning is adjusted in resonance with Brillouin acoustic wave the liquid moves as if it were confined inside helical channel having  $\lambda/2 = \pi/k$  pitch, radius  $D_0$  being equal to those of LG beam and the frequency detuning being equal to  $\delta\omega = 2nV_s\omega_{f,b}/c$ , where  $V_s$  is the speed of sound,  $\omega_{f,b}$  are the carrier frequencies of colliding vortices,  $c/n$  is speed of light in liquid. Typical spatial scales for this helical channel are  $D_0 \sim 10 - 100\mu m$ ,  $\lambda \sim 0.5 - 2\mu m$ .

The above helical flow at an ambient temperature  $\mathbf{T} = 300\mathbf{K}$  appears exactly in the case of the phase-conjugated reflection of the optical vortex  $E_f(\vec{r}, t)$  with orbital angular momentum per photon  $\ell\hbar$  from acoustic wave moving with speed  $V_s = \sqrt{(\partial p / \partial \rho)_S}$ . In this case a Stokes wave  $E_b(\vec{r}, t)$  with downshifted frequency appears [4]. This classical process has a quantum counterpart as a decay of a photon with orbital angular momentum  $\ell\hbar$  to the two particles: backward scattered Stokes photon with opposite OAM  $-\ell\hbar$  and forward corkscrew phonon with doubled vorticity  $2\ell$  [5].

For this micrometer-size helical channel width  $\lambda/2$  and diameter  $D_0$  the Reynolds number  $Re = \rho V_s \lambda / \eta$  is in the range  $10^{(2-3)}$  because of high value of velocity  $V_s \sim 10^5 cm/sec$ . This happens for the viscosities  $\eta \sim 10^{-(2-3)} poise$  of the most organic solvents and water based solutions at the room temperature used in applications of the Stimulated Brillouin scattering.

For  $Re = 10^{(2-3)}$  the flow in this channel must be indeed turbulent but electrostrictive pressure is strong enough to keep acoustic flow inside helical channel formed by a pair of isolated optical vortices. Moreover even the optical speckle field composed of a random set of intertwining optical vortices [6] drive the acoustic field into rotation exactly at the nodes of the optical interference pattern [5]. As a result, the acoustical turbulence induced by rotating multiply connected interference pattern of the incident optical speckle field  $E_f(\vec{r}, t)$  and phase-conjugated replica  $E_b(\vec{r}, t)$  is composed of the random set of vortex-antivortex pairs [4].

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## II. CLASSICAL PLASMA FLOW IN HELICAL CHANNEL INDUCED BY STIMULATED BRILLOUIN SCATTERING AT $keV$ TEMPERATURES

The laser plasma exhibit strong reflection of compressing radiation due to Stimulated Brillouin scattering [7]. We consider the ion-acoustic wave in an underdense plasma with temperature of electrons  $T_e \sim 10^6 K$  induced again by interference pattern of the two counterpropagating optical fields. As in the above described case of room temperature dielectric we presume that two phase-conjugated optical fields with slowly varying envelopes  $\mathcal{E}_f$  and  $\mathcal{E}_b$  generates ion-acoustic vortex  $\tilde{Q}(z, r, \phi, t)$  carrying doubled orbital angular momentum due to the motion in rotating helical channel. The SBS equations are:

$$\frac{\partial \mathcal{E}_f(z, r, \phi, t)}{\partial z} + \frac{n_r}{c} \frac{\partial \mathcal{E}_f}{\partial t} + \frac{i}{2k_f} \nabla_{\perp}^2 \mathcal{E}_f = \frac{i\Omega_p n_e}{4c n_c} \tilde{Q} \mathcal{E}_b \quad (3)$$

$$\frac{\partial \mathcal{E}_b(z, r, \phi, t)}{\partial z} - \frac{n_r}{c} \frac{\partial \mathcal{E}_b}{\partial t} - \frac{i}{2k_f} \nabla_{\perp}^2 \mathcal{E}_b = -\frac{i\Omega_s n_e}{4c n_c} \mathcal{E}_f \tilde{Q}^*, \quad (4)$$

and dimensionless slowly varying ion-acoustic wave complex amplitude  $\tilde{Q}$  is:

$$\begin{aligned} \frac{\partial \tilde{Q}(z, r, \phi, t)}{\partial z} + \frac{1}{c_{ia}} \frac{\partial \tilde{Q}}{\partial t} + \frac{2\gamma_{ia} \tilde{Q}}{c_{ia}} + \frac{i}{2(k_f + k_b)} \nabla_{\perp}^2 \tilde{Q} = \\ i(k_f + k_b) \mathcal{E}_f \mathcal{E}_b^* \frac{\epsilon_0}{2n_c k_B T_e}, \end{aligned} \quad (5)$$

where  $\nabla_{\perp} = (\partial_x, \partial_y)$ ,  $n_c \sim 10^{21} cm^{-3}$  is the critical plasma density. The resulting plasma flow in optically induced corkscrew channel has a form of the phase singularity with doubled vorticity  $2\ell$  [8]. The electron and ion currents proved to be large enough to produce magnetic dipole with kilogauss quasistatic magnetic field  $\tilde{\mathbf{B}}$ .

## III. HELICAL CHANNEL FILLED BY SUPERFLUID AT $\mu K$ TEMPERATURES

The macroscopic coherence of quantum fluid in multiply connected geometry leads to Hess-Fairbank effect [9]. The superfluid in annular cylindrical container rotating with low angular velocity  $\Omega_{\oplus} \sim 10^{-4}$  rad/sec is not dragged by rotating boundaries. This happens when liquid  $^4He$  is cooled below critical temperature  $T_{\lambda}$  and angular momentum  $\langle \hat{L}_Z \rangle \sim N\ell\hbar$  (rotational energy per particle is  $\langle E_{rot} \rangle \sim \ell^2 \hbar^2 / 2mR^2$ ) of superfluid is much smaller than those of classical liquid  $L_Z \sim NmR^2\Omega$ , where  $R$  is a mean radius of flow,  $m$  is atom mass,  $N$  is a number of atoms in a rotating ensemble,  $\ell$  is a winding number [10, 12].

The interesting analog of the Hess-Fairbank effect may be proposed when superfluid is placed in container having corkscrew shape rather than cylindrical one [11]. For this purpose not only liquid  $^4He$  is feasible but microkelvin trapped alkali gases are suitable as well. The Gross-Pitaevskii equation is applicable in the latter case. The trapping helical optical potential with soft penetrable walls is as follows:

$$U_{opt}(z, r, \theta, t) \sim \frac{U_0 \cdot r^{2|\ell|}}{(1 + z^2/(k^2 D_0^4))} \exp \left[ -\frac{2r^2}{D_0^2(1 + z^2/k^2 D_0^4)} \right] [1 + \cos(2kz + 2\ell\theta + \delta\omega t)], \quad (6)$$

where  $\delta\omega$  is angular Doppler shift induced by rotation of the reference frame or emulated by rotation of Dove prism in a phase conjugated setup [3]. Transformation to the reference frame rotating synchronously with angular velocity  $\Omega = \delta\omega/2\ell = |\vec{\Omega}_{\oplus}|$  with trapping helix leads to the time-dependent Gross-Pitaevskii equation (GPE) [13–15]:

$$i\hbar \frac{\partial \Psi}{\partial t} = -\frac{\hbar^2}{2m} \Delta \Psi + \tilde{U}_{opt}(z, r, \theta) \Psi + g|\Psi|^2 \Psi - \Omega \hat{L}_z \Psi, \quad (7)$$

where the stationary wavefunctions for the superfluid ensemble  $\Psi = \Phi(z, r, \theta) \exp(-i\mu t/\hbar)$  are given by:

$$\mu \Phi = -\frac{\hbar^2}{2m} \Delta \Phi + \tilde{U}_{opt}(z, r, \theta) \Phi + g|\Phi|^2 \Phi + \Omega \hbar \frac{\partial \Phi}{\partial \theta}, \quad g = \frac{4\pi \hbar^2 a_S}{m}, \quad (8)$$

where  $a_S$  is  $S$ -wave scattering length. We evaluate linear and angular momenta of the superfluid ensemble in a helical container [14] and discuss the possibilities of rotations detection with this geometry. Noteworthy the "observer" velocity vector  $\vec{V}$  with respect to "lab frame" has two components [8]: the azimuthal velocity  $\vec{V}_{\theta} = \vec{\Omega}_{\oplus} \times \vec{r}$  stands for helix rotation around LG propagation axis  $\vec{z}$  while helix pitch velocity  $\vec{V}_z = (\vec{z}/z)\delta\omega/2k$  is responsible for wavetrain translation along  $\vec{z}$  [16].

#### IV. DISCUSSION

It is shown in the first two examples that certain classical liquids in helical container are completely dragged by rotating boundaries, so that observer placed in the reference frame collocated with rotating container will not detect rotation. On the contrary the quantum fluid placed in slowly rotating container will remain in rest in laboratory frame and it will move translationally from the point of view of observer placed in rotating frame collocated with dragged classical fluid. Experimentally the micrometer size corkscrew channels may be realized as interference patterns of detuned optical vortices (for a trapped degenerate quantum gas) or as a twisted glass pipe (for a  $^4\text{He}$  cooled below  $\lambda$ -point) [17–19].

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